VRSVFMF 2026-03-18 zamudil 30 min.

N

...

Rodovné fje

$G_X(s) = e^{s+s^2-2}$

$P(X=2) = ?$

$G'_X(s) = e^{s+s^2-2}(1+2s)$

$G''_X(s) = e^{s+s^2-2}(1+2s)(1+2s) + e^{s+s^2-2} \cdot 2$

$$P(X=2) = \frac{e^{-2}+2e^{-2}}{2!} = \frac{3}{2}e^{-2} \approx 0{,}203$$

alternativno

$$G_X(s) = e^{-2}\left(1 + (s+s^2)^2 + \frac{(s+s^2)^2}{2!} + \frac{(s+s^2)^3}{3!} + \cdots\right) =$$

$$= e^{-2}\left[1 + s + s^2 + \frac{1}{2}s^2 + s^3 + \frac{1}{2}s^4 + \frac{1}{6}s^3 + \frac{1}{2}s^4 + \frac{1}{2}s^5 + \frac{1}{6}s^6 + \cdots\right] =$$

(same visje potence)

$$= e^{-2}\left(1 + s + \frac{3}{2}s^2 + \cdots\right)$$

$G_X(s) = E(s^X)$

$G_X(1) = 1$

$G_X^{(k)}(s) = E\left(X(X-1)\cdots(X-k+1)s^{X-k}\right)$

$G_X^{(k)}(1) = E\left(X(X-1)\cdots(X-k+1)\right)$

k-ti faktorski moment

k-ti začetni moment: $E(X^k)$

$X \sim Geo(p)$:

$P(X=k) = p(1-p)^{k-1}$; $k = 1, 2, 3, \ldots$

$G_X(s) = \sum_{k=1}^{\infty} p(1-p)^{k-1}s^k = \ldots = \frac{ps}{1-(1-p)s}$

$E(X) = ?$

$D(X) = ?$ ...

N

$X \sim Geo(1/2)$ $\quad Y \sim Geo(1/3)$ $\quad X+Y \sim ?$

$G_X(s) = \frac{1/2 \cdot s}{1-1/2\,s} = \frac{s}{2-s}$ $\quad G_Y(s) = \frac{1/3 \cdot s}{1-1/3\,s} = \frac{s}{3-2s}$ $\quad G_{X+Y}(s) = \frac{s}{2-s} \cdot \frac{s}{3-2s} =$

$= \frac{s^2}{(2-s)(3-2s)} = s^2 \frac{1}{(2-s)(3-2s)}$

$$\frac{1}{(2-s)(3-2s)} = \frac{A}{2-s} + \frac{B}{3-2s} = \frac{(3-2s)A + (2-s)B}{(2-s)(3-2s)}$$

$$1 = 3A - 2sA + 2B - sB = 3A + 2B - s(2A+B)$$

$2A+B=0$ $\quad B = -2A = 2$

$3A+2B=1$ $\quad A = -1$

$3A - 4A = 1$

$-A = 1$

$$G_{X+Y}(s) = s^2 \left(\frac{2}{3-2s} - \frac{1}{2-s} \right) = \dots = \sum_{n=2}^{\infty} \left(\left(\frac{2}{3}\right)^{n-1} - \left(\frac{1}{2}\right)^{n-1} \right) s^n$$

$\Rightarrow n = 2,3,4 \qquad P(X+Y=n) = \left(\frac{2}{3}\right)^{n-1} - \left(\frac{1}{2}\right)^{n-1}$